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Measuring tax progressivity:

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S. Subramanian[1]

Abstract

In measuring tax progressivity, resort is mainly had to the indices associated with the names of Kakwani (1977) and Suits(1977) respectively. While both measures are plausible ones, it would help, in discriminating between the two, to have a handle on their respective properties and on the analytical connection between the two. The work of Formby, Seaks and Smith (1981) greatly facilitates such an understanding. Building further on this work, the present article emphasizes the view that a generalized measure of tax progressivity can be written as a weighted sum of income-specific gaps between cumulative income shares and cumulative tax shares. Whether a given gap is accorded more, less or the same weight as income increases could be taken to be a reflection of, respectively, 'conservative', 'liberal', or 'neutral' values regarding the progressivity of a tax system. This paper suggests that Kakwani's measure is a neutral one, and Suits' measure a conservative one; it extends these two indices to a third, liberal, measure, which is shown to be a weighted average of the Kakwani and Suits indices. These indices are shown to be special cases of an entire parametrized family of tax progressivity measures.

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MEASURING TAX PROGRESSIVITY:

A SYNTHESIS AND EXTENSION

1. Introduction

Two of the most commonly employed measures of tax progressivity are those proposed, more or less simultaneously, by Nanak Kakwani (1977) and David Suits (1977). The idea, in both cases, is to evaluate the overall extent of progressivity of an income-tax schedule in terms of an aggregation of income-specific deviations of the tax-schedule from a pattern of strict proportionality. Putting it differently, the idea is to measure tax progressivity by aggregating, at each level of income, the gap (positive or negative)—call this the 'proportionality gap'--between the cumulative share in income earned and the cumulative share in taxes paid. In a very helpful analysis of the respective characteristics of the Kakwani and Suits measures of progressivity, it was pointed out by John Formby, Terry Seaks and James Smith (1981) that the two measures differ, in their respective schemes of aggregation of the income-specific gaps just alluded to, in terms of the weights on the gaps implicitly employed by them. This paves the way for a general formulation of a progressivity measure as a weighted sum of gaps between cumulative income-shares and cumulative tax-shares, where the weights reflect one's normative assessment of how, if at all, a ' proportionality gap' of given magnitude should vary with the level of income at which it occurs. If a measure displays a system of equal weights, one might call it a 'neutral' measure; and a 'conservative' (respectively, 'liberal') measure if the weights increase (respectively, decline) with the level of income. A great merit of the Formby et al. (1981) paper is that it enables one to see that the Kakwani measure of tax progressivity is a neutral one, while the Suits measure is a conservative one. This paves the way for a simple extension of the essential idea behind the Kakwani and Suits measures to the formulation of a liberal measure of tax progressivity. This last measure, as it happens, turns out to be a simple weighted average of the Kakwani and Suits indices. The rest of the paper is devoted to an elaboration of the preceding overview.

2. Notation, Definitions, and Indices

Much of the notation employed in what follows is due to Kakwani (1980). x is a continuous random variable signifying income, and is distributed over the interval $[x_0, x^*] \equiv I$, with $x_0 \geq 0$ and $x^* < \infty$. The density function of x, $f(x)$, denotes the probability of x falling in the interval $[x, x+dx]$, while the cumulative distribution function, $F(x)$, is the proportion of the population with incomes less than x:

$$(1)\quad F(x) = \int_{x_0}^{x} f(u)du \quad \forall x \in I.$$

The first-moment income-distribution function of x, $F_1(x)$, is the cumulative income-share of those with incomes less than x:

$$(2)\quad F_1(x) = (1/\mu)\int_{x_0}^{x} uf(u)du \quad \forall x \in I,$$

where $\mu \equiv \int_I xf(x)dx$ is the mean of the distribution. Throughout this paper, attention will be confined—for reasons that will be clarified later—to distributions which are at least a little unequal, that is, to distributions for which it is true that $x^* > \mu$.

The *Lorenz curve* is the plot of $F_1(x)$ against $F(x)$ in the unit square, and the slope of the Lorenz curve at any level of income x is given by

$$(3)\quad dF_1(x)/dF(x) = F_1'(x)/F'(x) = [xf(x)/\mu]/f(x) = x/\mu \quad \forall x \in I.$$

For all $x \in I$, let $t(x)$ be the tax paid by a unit with income x. The average tax paid is denoted by $\bar{t} \equiv \int_I t(x)f(x)dx$. $F_1^T(x)$ is the cumulative share in taxes paid by those with incomes less than x:

$$(4)\quad F_1^T(x) = (1/\bar{t})\int_{x_0}^{x} t(u)f(u)du \quad \forall x \in I.$$

The *tax concentration curve* is the plot of $F_1^T(x)$ against $F(x)$ in the unit square.

The *Gini inequality index G* is a measure of inequality in the distribution of incomes, and is given—this is a standard result—by the normalized area distance between the diagonal of the unit square and the Lorenz curve:

$$(5)\quad G = 2\int_I [F(x) - F_1(x)]dF(x).$$

The *tax-concentration index* C_T is a measure of disproportionality in the distribution of taxes paid, and is given, in an expression analogous to that for the Gini coefficient of income inequality, by:

$$(6)\quad C_T = 2\int_I [F(x) - F_1^T(x)]dF(x).$$

The *Kakwani measure of tax progressivity* P_K is just the difference between the values of the tax-concentration and Gini measures:

$$(7)\quad P_K = C_T - G = \int_I 2[F_1(x) - F_1^T(x)]dF(x).$$

What one may call a *Suits concentration curve* is derived by plotting the cumulative share in taxes paid, $F_1^T(x)$, against the cumulative share in income, $F_1(x)$, in the unit square; and the

Suits measure of tax progressivity P_S is just the normalized area distance between the diagonal of the unit square and the Suits concentration curve:

(8) $P_S = \int_I 2[F_1(x) - F_1^T(x)]dF_1(x).$

Noting that $dF_1(x) = [dF_1(x)/dF(x)]dF(x)$, and substituting for $dF_1(x)/dF(x)$ from (3), one has: $dF_1(x) = (x/\mu)dF(x),$ whence (8) can be written as:

(9) $P_S = \int_I (x/\mu)[2\{F_1(x) - F_1^T(x)\}]dF(x).$

Equation (9) is, essentially, the expression for the Suits progressivity measure as derived by Formby et al (1981).

3. Weights and Measures

The central point of the present article is contained in this observation of Formby et al's (1981; p.1018): '…the Kakwani and Suits measures differ by [a] weighting factor…'. This observation paves the way for the proposition that a plausible measure of tax progressivity P can be written as a weighted average of twice the income-specific proportionality gaps (or differences between cumulative income shares and cumulative tax shares):

(10) $P = \int_I w(x)[2\{F_1(x) - F_1^T(x)\}]dF(x),$

where $w(x) \geq 0 \ \forall x$ and $\int_I w(x)dF(x) = 1.$

In (10), $w(x)$ is the weight attached to twice the proportionality gap $F_1(x) - F_1^T(x)$ which obtains at the income level x, and from the expression for the Kakwani progressivity measure provided in (7), one can see that the weight $w_K(x)$ attached to $2\{F_1(x) - F_1^T(x)\}$ is given by $w_K(x) = 1,$ so that $w_K'(x) = 0 \ \forall x \in I.$ Similarly, from the expression for the Suits measure provided in (9), one can see that the weight $w_S(x)$ attached to twice the proportionality gap which obtains at the income level x is given by $w_S(x) = x/\mu,$ so that $w_S'(x) = 1/\mu > 0 \ \forall x \in I.$ (It is easily verified that $\int_I w_K(x)dF(x) = \int_I w_S(x)dF(x) = 1.$)

Given the expression in (10) for a general tax progressivity measure P, the question arises: is there a case for making explicit the normative significance of the sign of $w'(x)$? It seems fair to suggest that a 'liberal' view of tax progressivity would require that in a measure of the type captured in (10), a given proportionality gap should have a greater salience the *lower* the level of income at which the gap occurs: this would argue the case for a system of weights which *decline* with income. Seen in this light, a progressivity measure in which $w'(x)$ is zero could be called a '*neutral*' measure; a measure in which $w'(x)$ is positive could be called a

'*conservative*' measure; and a measure in which $w'(x)$ is negative could be called a '*liberal*' measure. By this reckoning, the Kakwani measure P_K is a neutral measure, while the Suits measure P_S is a conservative one.

A simple means of obtaining a liberal measure of progressivity would be to have a weighting function in (10) which is (for example) of the type $w(x) = \left(\frac{x^*-x}{x^*-\mu}\right) \ \forall x \in I.$ Note first that $\left(\frac{x^*-x}{x^*-\mu}\right)$ is positive for all x in view of the preliminary assumption in the preceding section which confines attention to distributions which are at least a little unequal. For note that, if the distribution is a perfectly equal one, then $x^* = \mu,$ and $w(x^*)$ would result in the problematic quantity of 0/0, which is avoided by considering only distributions for which $x^* > \mu.$ Second, clearly $\int_I \left(\frac{x^*-x}{x^*-\mu}\right) dF(x) = 1.$ Third, when $w(x) = \left(\frac{x^*-x}{x^*-\mu}\right),\ w'(x) = -[1/(x^*-\mu)] < 0 \ \forall x \in I.$

By setting $w(x) = \left(\frac{x^*-x}{x^*-\mu}\right) \forall x \in I$ in the right hand side of (10), one obtains a liberal measure of tax progressivity which one may call P_{KS}, where the subscript 'KS' signals the fact that, as it happens, the measure is a combination of the Kakwani and Suits measures P_K and P_S respectively. To see this, note (given (10)) that

$$P_{KS} = \int_I \left(\frac{x^*-x}{x^*-\mu}\right)[2\{F_1(x) - F_1^T(x)\}]dF(x)$$

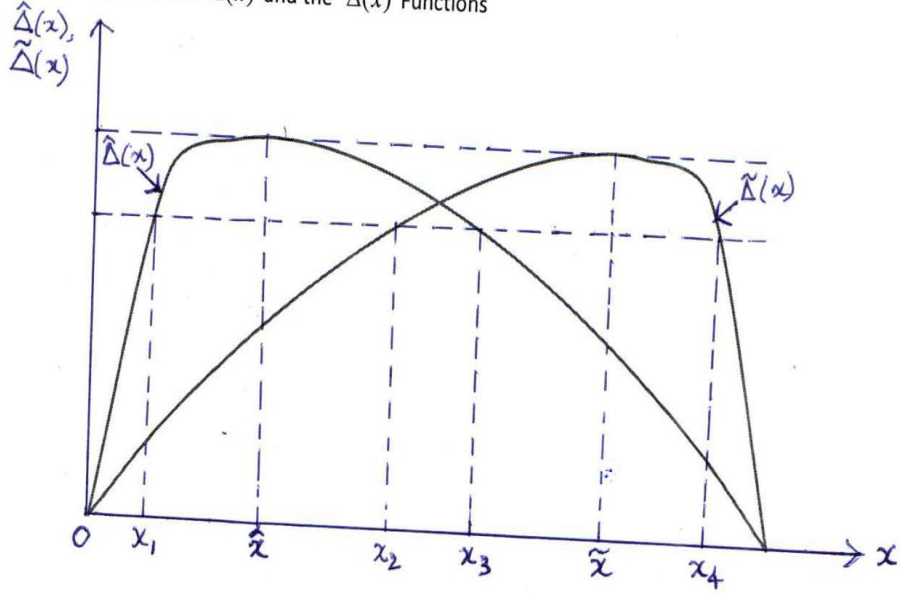
$$= \left(\frac{x^*}{x^*-\mu}\right)\int_I [2\{F_1(x) - F_1^T(x)\}]dF(x) - \left(\frac{\mu}{x^*-\mu}\right)\int_I (x/\mu)[2\{F_1(x) - F_1^T(x)\}]dF(x).$$

Given the expressions for P_K and P_S in (7) and (9) respectively, the preceding equation can be written as

$$(11) \qquad P_{KS} = \left(\frac{x^*}{x^*-\mu}\right)P_K - \left(\frac{\mu}{x^*-\mu}\right)P_S.$$

If we set $\left(\frac{x^*}{x^*-\mu}\right) \equiv \alpha (> 0),$ then (11) can be written as: $P_{KS} = \alpha P_K + (1-\alpha)P_S$: P_{KS} turns out to be a weighted average of P_K and P_S. It may be added that for a distribution in which income is inordinately concentrated in the hands of the richest person, so that $x^* >> \mu,$ the measure P_{KS} will converge on the measure P_K (as (11) should make clear).

Figure 1: The $\hat{\Delta}(x)$ and the $\tilde{\Delta}(x)$ Functions



For each of the measures considered, one may take the tax system to be overall neither progressive nor regressive if the value of the measure is zero; progressive if the value of the measure is positive; and regressive if the measure is negative. Which measure one chooses to employ could have practical implications for the ranking of alternate tax systems by progressivity: this ranking could vary according to the precise measure employed. To see this more clearly, consider the following.

Imagine two income distributions $\hat{F}(\cdot)$ and $\widetilde{F}(\cdot)$ respectively, which share the same mean μ. Let $\hat{\Delta}(x) \equiv \hat{F}_1(x) - \hat{F}_1^T(x)$ and $\widetilde{\Delta}(x) \equiv \widetilde{F}_1(x) - \widetilde{F}_1^T(x)$ $\forall x \in I$. Suppose that each of $\hat{\Delta}(x)$ and $\widetilde{\Delta}(x)$ is positive for all $x \in (0, x^*)$, so that each of $\hat{\Delta}(x)$ and $\widetilde{\Delta}(x)$ first rises from zero at $x = 0$, peaks at some level of income ($\hat{x}$ for the former function and $\widetilde{x}$ for the latter), and then declines to zero at $x = x^*$. Suppose further that (i) $\hat{\Delta}(x)$ is skewed to the right while $\widetilde{\Delta}(x)$ is skewed to the left; (ii) the $\widetilde{\Delta}(x)$ function is a mirror-image of the $\hat{\Delta}(x)$ function; and (iii) the areas under the two curves are the same. All of this is captured in Figure 1.

Now consider four distinguished levels of income x_1, x_2, x_3 and x_4, such that—see Figure 1--$\hat{\Delta}(x_1) = \widetilde{\Delta}(x_2) = \hat{\Delta}(x_3) = \widetilde{\Delta}(x_4)$. The tax progressivity measure P_K weights $\Delta(x)$ identically at all levels of income, so the weighted value of $\hat{\Delta}(x_1)$ is the same as that of $\widetilde{\Delta}(x_2)$, just as the weighted value of $\hat{\Delta}(x_3)$ is the same as that of $\widetilde{\Delta}(x_4)$. This property would hold for every income level in the interval $[0, \hat{x}]$, $\hat{x}$ being the level of income at which $\hat{\Delta}(x)$ peaks. It can be seen from this that under the weighting scheme characterising the P_K measure, the area under the weighted $\hat{\Delta}(x)$ function must be the same as that under the weighted $\widetilde{\Delta}(x)$ function: that is, the P_K measure of progressivity must be the same for the respective tax regimes in both the scenarios considered.

Next consider the P_S measure. The weighting scheme underlying this measure attaches a weight of x/μ to $\Delta(x)$ for all x: so the weight x_1/μ on $\hat{\Delta}(x_1)$ is less than the weight x_2/μ on $\widetilde{\Delta}(x_2)$, just as the weight x_3/μ on $\hat{\Delta}(x_3)$ is less than the weight x_4/μ on $\widetilde{\Delta}(x_4)$. This property holds for every income level in the interval $[0, \hat{x}]$, implying that given the weighting system underlying the P_S measure, the area under the weighted $\hat{\Delta}(x)$ function must be less than that under the weighted $\widetilde{\Delta}(x)$ function: that is, the P_S measure of progressivity must be less for the the '^' regime than for the '~' regime.

Repeating the preceding paragraph word for word except for the required contextual variations, one has the following. For the P_{KS} measure, the underlying weighting scheme attaches a weight of $(x^* - x)/(x^* - \mu)$ to $\Delta(x)$ for all x: so the weight $(x^* - x_1)/(x^* - \mu)$ on $\hat{\Delta}(x_1)$ is more than the weight $(x^* - x_2)/(x^* - \mu)$ on $\widetilde{\Delta}(x_2)$, just as the weight $(x^* - x_3)/(x^* - \mu)$ on $\hat{\Delta}(x_3)$ is more than the weight $(x^* - x_4)/(x^* - \mu)$ on $\widetilde{\Delta}(x_4)$. This property holds for every income level

in the interval $[0,\hat{x}]$, implying that given the weighting system underlying the P_{KS} measure, the area under the weighted $\hat{\Delta}(x)$ function must be greater than that under the weighted $\tilde{\Delta}(x)$ function: that is, the P_{KS} measure of progressivity must be larger for the '^' scenario than for the '~' scenario.

Briefly, and employing obvious notation, one has: $\hat{P}_K = \tilde{P}_K$; $\hat{P}_S < \tilde{P}_S$; and $\hat{P}_{KS} > \tilde{P}_{KS}$, so the rankings of the two tax systems under the '^' and the '~' regimes are different for the three measures: both P_S and P_{KS} deny the identicalness of ranking implied by P_K, and in doing so, P_S and P_{KS} reverse each other's ranking. In pronouncing on questions of more and less, one cannot be indifferent to the choice of measure.

4. A Family of Tax Progressivity Measures

The tax progressivity measures P_K, P_S and P_{KS} are actually special cases of a general class of measures P_λ, parametrized by the scalar λ, which can be specified along the following lines. First, define the sets of real numbers $\mathcal{D}_1 \equiv (-\infty,0]$, $\mathcal{D}_2 \equiv [1,\infty)$, and $\mathcal{D} \equiv \mathcal{D}_1 \cup \mathcal{D}_2$. The set of numbers contained in $\mathcal{D}$ is the entire real line with just the open interval (0,1) excluded. The P_λ family of tax progressivity measures can now be defined as:

$$(12) \qquad P_\lambda = \int_I w_\lambda(x)[2\{F_1(x) - F_1^T(x)\}]dF(x); \;\; w_\lambda(x) = \left(\frac{\lambda x^* - x}{\lambda x^* - \mu}\right) \;\; \forall x \in I, \forall \lambda \in \mathcal{D}.$$

It bears pointing out that P_λ is not defined for values of λ in the interval (0,1): for positive fractional values of λ, the weighting function $\left(\frac{\lambda x^* - x}{\lambda x^* - \mu}\right)$ can be negative, which is why the range of values permitted for λ to take is the real line *excluding* the interval (0,1), namely the set $\mathcal{D}$ defined earlier.

Notice, given (8), (9) and (11) respectively, that

$$(13) \qquad Lim_{\lambda\uparrow\infty} P_\lambda = Lim_{\lambda\downarrow-\infty} P_\lambda = \int_I 2[F_1(x) - F_1^T(x)]dF(x) \equiv P_K;$$

$$(14) \qquad P_1 = \int_I \left(\frac{x^* - x}{x^* - \mu}\right)[2\{F_1(x) - F_1^T(x)\}]dF(x) \equiv P_{KS}; \text{ and}$$

$$(15) \qquad P_0 = \int_I (x/\mu)[2\{F_1(x) - F_1^T(x)\}]dF(x) \equiv P_S.$$

One can see that P_1 (or P_{KS}) is the most 'liberal' measure in the P_λ family, and that as λ increases from 1, one has a sequence of decreasingly liberal measures until, in the limit, as λ goes to infinity, the measure converges on the 'neutral' Kakwani index. Similarly, P_0 (or the Suits measure P_S) is the most 'conservative' measure in the P_λ family, and as λ declines from

0, one has a sequence of decreasingly conservative measures until, in the limit, as λ goes to minus infinity, the measure again converges on the 'neutral' Kakwani index. As λ moves rightward from 1 to $+\infty$, we have a transition of measures from liberal to neutral; and as λ moves leftward from 0 to $-\infty$, we again have a transition of measures, this time from conservative to neutral. Over the range of values that can be assumed by λ, and the corresponding tax progressivity measures associated with these values, three convenient stopping places are the 'neutral' Kakwani measure (P_K), the 'conservative' Suits measure (P_S), and the 'liberal' 'Kakwani-Suits' measure (P_{KS}).

5. Concluding Observations

The Kakwani (1977) and Suits (1977) indices are the best known and most widely used measures of tax progressivity in the literature on the subject. In an instructive intervention, Formby et al (1981) have discussed the properties of the two indices and identified the analytical relation between them. In particular, their work points to the plausibility of viewing a measure of tax progressivity as a weighted aggregation of income-specific 'proportionality gaps' (that is, income-specific deviations between cumulative income shares and cumulative tax shares). The present paper suggests that how these weights are assigned could be taken to reflect one's values in a particular sense of the notions of 'neutrality', 'conservatism' and 'liberalism'. More concretely, if the weights in question are constant across income levels, the suggestion is that any given proportionality gap is assigned the same significance irrespective of the level of income at which it occurs; if the weights decline (respectively, increase) with income, the suggestion would be that one accords greater significance to a given proportionality gap the lower (respectively, higher) the income level at which the gap occurs. These might be seen to correspond to value judgements on tax progressivity which are, respectively, 'neutral', 'liberal', and 'conservative'. As it turns out, the Kakwani measure is a neutral one, and the Suits measure a conservative one. A natural extension would be to a liberal measure: this article has proposed such a measure, which, as it happens, is a weighted average of the Kakwani and Suits measures, and may be called a 'Kakwani-Suits index'. The paper also advances an entire class of progressivity measures, of which three special cases are the Kakwani, the Suits, and the Kakwani-Suits measures. It cannot be a matter of indifference as to which of these measures is pressed into service, since the rankings of tax systems by progressivity are not invariant with respect to the measure employed. This is why it is important to recognize that a question of choice regarding measures must be addressed, through clarity about the value judgements that implicitly underlie the choice.

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